Vibrational Relaxation in Gas-Particle Nozzle Flows: A Study of the Effect of Particles

R. K. Thulasiram* and N. M. Reddy†
Indian Institute of Science, Bangalore 560 012, India

Introduction

NONEQUILIBRIUM nozzle flows of pure gases is common in many aerospace applications such as rocket nozzles, and hypersonic simulation facilities such as wind-tunnel/shock-tunnel nozzles. The basic problems, including the problem of obtaining numerical solutions for nozzle flows with vibrational energy relaxation, have been studied by several authors.[1–8] With the advent of metallized propellants for solid rocket motors, the nozzle flow consists not only of pure gases. The metallized propellant burnt in the combustion chamber becomes metal oxide particles and are dragged out of the combustion chamber by the nozzle expansion process of the gas. Arc-heated test gas in hypersonic wind tunnels also contains large numbers of particles flowing with the gas.[9,10] It is a known fact from earlier studies that the presence of particles in the flow can modify the flow expansion process. The effects of such particles on the internal energy nonequilibrium phenomena during the nozzle expansion process is studied in this investigation.

Governing Equations

The expansion of a diatomic gas (such as nitrogen) in a nozzle is considered such that vibration is excited and dissociation is negligible. Reservoir conditions are employed which correspond to the lower range of temperatures found in hot shot and shock tunnels, in steady-state nitrogen tunnels, and in gasdynamic lasers. The energy in vibrational modes is small compared to that in dissociation and ionization at high temperatures. However, this small amount of internal energy is adequate to produce nontrivial gasdynamic effects if appreciable departure from equilibrium (nozzle) flow occurs. This vibrational nonequilibrium can produce effects that are of much importance on nozzle test flow conditions in the high-speed wind-tunnel facilities. For nonequilibrium (nonequilibrium between gas and particles and internal energy nonequilibrium within the gas) flow condition in two-phase nozzle flows, the computation of the flow variables requires a rate expression for the vibrational relaxation process in addition to the usual governing equations for gas-particle mixture. The rate expressions for vibrational energy relaxation is given as[3]

$$\frac{\mathrm{d}(e_v')}{\mathrm{d}t'} = \frac{1}{\tau'}\left[(e_v')^{\mathrm{eq}} - (e_v')\right] \tag{1}$$

where τ' is the vibrational relaxation time for a pure diatomic gas. This equation has to be solved simultaneously with the governing equations and the additional variable is the vibrational energy (e_v) of the gas.

Analysis of the gas-particle flow in a nozzle is complex because of the large number of parameters involved and the computations become laborious if the effects of different parametric conditions of the problem are required. However, a single correlating parameter has been obtained earlier[11,12] combining many of the important parameters by transforming the governing equations appropriately. In the present investigation the same method has been applied to study the effect of phase nonequilibrium on the internal energy relaxation phenomena of a diatomic gas. The procedure can be summarized as follows. A new variable $\alpha[= -\ln(\rho)]$ is introduced and the equations are transformed with α as the independent variable. In order to obtain nonequilibrium solutions, equilibrium initial conditions are imposed on these transformed equations. The reservoir entropy S_0 is shown[12] to enter as an additional parameter in equilibrium solutions. To reduce the number of parameters to a minimum, another transformation is introduced and general correlating parameters are deduced. The final set of equations with $\xi(= S_0 - \alpha)$ as the independent variable are given by

$$u\frac{\mathrm{d}u}{\mathrm{d}\xi} + \eta u\frac{\mathrm{d}u_p}{\mathrm{d}\xi} + \theta_N\frac{\mathrm{d}\psi}{\mathrm{d}\xi} + \theta_N\psi = 0 \tag{2}$$

$$u\frac{\mathrm{d}u}{\mathrm{d}\xi} + \left(\frac{7}{2}\right)\theta_N\frac{\mathrm{d}\psi}{\mathrm{d}\xi} + G\theta_N\frac{\mathrm{d}\phi}{\mathrm{d}\xi} + \eta\left(u_p\frac{\mathrm{d}u_p}{\mathrm{d}\xi} + \frac{\gamma}{\gamma - 1}\delta\theta_N\frac{\mathrm{d}\psi_p}{\mathrm{d}\xi}\right) = 0 \tag{3}$$

$$\frac{\mathrm{d}u_p}{\mathrm{d}\xi} = -\frac{e^{\chi - \xi/ij}}{N_s}\frac{u - u_p}{u_p}(uZ)^{-1/ij} \tag{4}$$

$$\frac{\mathrm{d}\psi_p}{\mathrm{d}\xi} = -\frac{e^{\chi - \xi/ij}}{N_s}\frac{1}{\delta}\frac{\psi - \psi_p}{u_p}(uZ)^{-1/ij} \tag{5}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}\xi} = -\frac{\psi}{N_s}\{\exp[\chi_I + \xi(1 - 1/ij) - Y\psi^{-1/3}]\}\left[\frac{(e_v)^{\mathrm{eq}} - e_v}{G}\right] \tag{6}$$

where

$$G = \frac{e^{1/\phi}(1/\phi)^2}{(e^{1/\phi} - 1)^2} - \frac{e^{N/\phi}(N/\phi)^2}{(e^{N/\phi} - 1)^2}$$

$$\chi = \ln\left[\frac{(\rho_* u_*)^{1/ij}}{(ij)\tau_v}\right] + S_0/ij$$

$$\chi_I = \ln\left[\frac{p_0' L' \theta_N (\rho_* u_*)^{1/ij}}{(ij)u_0' W}\right] - S_0(1 - 1/ij)$$

$$N_s = \frac{M_v^2}{M_v^2 - 1}(1 - A_v^{-1/i})^{(j-1)/j}$$

where standard notations are used for the flow variables. The subscripts p and * refer to particles and nozzle throat condition, respectively, if not specified otherwise; superscript eq refers to equilibrium condition; ( ) refers to dimensional quantities; σ_p is the mass fraction of the particles in the mixture; η is the loading ratio [mass flow rate of the particles $(\dot{m}_p)$ to that of gas $(\dot{m})$]; δ is the ratio of the specific heat of the particles and gas; τ_v is the velocity relaxation time of particles $[=\rho_p d_p^2/(18\mu)]$; d_p is the diameter of the (spherical) particles; and μ is the viscosity of the gas phase. The reservoir quantities, nozzle scale parameter L', and the nozzle throat area have been used for nondimensionalization; the quantities $u_0' = (RT_0')^{0.5}$ and $p_0' = \rho_0' u_0'^2$ and $h_0' = RT_0'$ are used for nondimensionalizing velocity, pressure, and enthalpy, respectively. A_v is the virtual area ratio given by $A_v = A/Z$; Z is a parameter representing phase nonequilibrium character-

Received Aug. 18, 1992; revision received Dec. 11, 1992; accepted for publication Jan. 7, 1993.

*Research Scholar, Department of Aerospace Engineering; currently Post-Doctoral Fellow, Department of Mechanical Engineering, Concordia University, Montreal, H3G 1M8, Canada. Member AIAA.

†Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

istics of the flow given by $Z = 1 + \eta u_p/u$; M_v is virtual Mach number given by $M_v = u/a_v$ corresponding to virtual speed of sound a_v given by $a_v = [(1/Z)(dp/d\rho)]^{1/2}$ and p is the pressure; i and j are the nozzle-shape parameters with $i = 1, j = 2$ representing hyperbolic nozzle and $i = 2, j = 1$ representing conical nozzle; N is the population density of the energy levels of the gas and W ($= 2.55 \times 10^{-8}$ atm-s) is a constant. In the present analysis, the nondimensionalized temperatures are normalized with reference to the characteristic vibrational temperature θ_N of nitrogen such that ψ and ϕ are the normalized translational and vibrational temperatures of the gas phase; and ψ_p is the normalized temperature of the particle phase. An examination of this set of governing equations reveals that for a given value of (ij) and a given gas, the solution will depend on two parameters χ_l and χ, the first one characterizing the vibrational relaxation of the diatomic gas and the second characterizing the particle properties. Simple expressions for the functions $f(Re)$ and $Nu(Re)$ have been used here. The function N_s has been correlated[12] accordingly and has been used for the current investigation.

Results and Discussion

The final set of equations [Eqs. (2–6)] have been solved for the variables u, u_p, ψ, ψ_p, and ϕ using Runge-Kutta-Gill method, and the results are presented in the form of graphs. The vibrational energy has been assumed to be in equilibrium with the translational and rotational energy modes for the flow from the nozzle reservoir up to the nozzle throat. The conditions at the throat are calculated from the equilibrium solution and serve as the initial conditions for the nonequilibrium calculations. Though the results are presented in the transformed (ξ) plane, they can easily be transformed into physical plane using the definition of (ξ). As mentioned earlier, the parameters χ and χ_l contain all the important parameters of the problem, and hence, control the characteristics of the solutions. The parameter χ which controls the particle characteristics (through τ_v) introduced in Eqs. (4) and (5) is given by $\chi = \ell n\{[(\rho_* u_*)^{1/ij}]/[(ij)\tau_v]\} + S_0/ij$. It can be inferred that small value of χ corresponds to particles of large size and vice versa. It is known from earlier investigation on gas-particle flows that large particles (which correspond to small χ) present in the flow freeze at the initial conditions and therefore have no effect on the gas, whereas the small particles (which correspond to large χ) are almost in equilibrium with the gas during the expansion process and show pronounced effects on the gas. Hence, the variation in the value of χ scans the complete nonequilibrium (between gas and particles) flow regime from equilibrium to frozen conditions.

Figure 1 shows the profiles of the translational temperatures ψ and vibrational temperatures ϕ along the nozzle starting

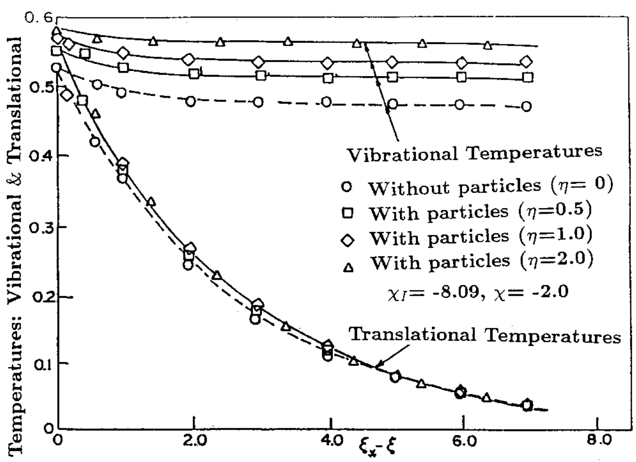

Fig. 1 Translational and vibrational temperature profiles along the nozzle axis.

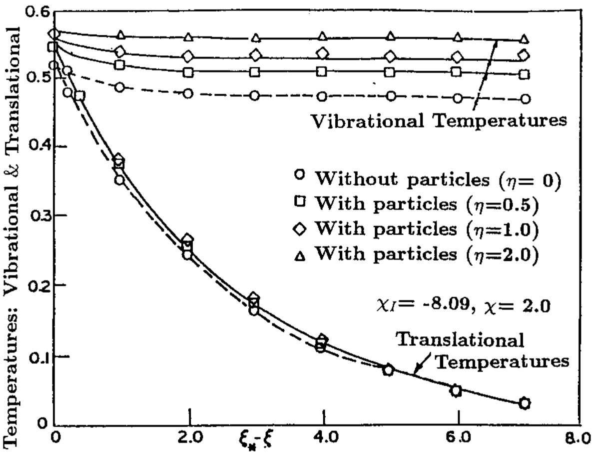

Fig. 2 Translational and vibrational temperature profiles along the nozzle axis.

from the nozzle throat for different η and for a given χ value. The physical phenomenon that the vibrational energies relax at a slower rate than the translational energy during the nozzle expansion process could be observed from this figure. The vibrational temperature tries to equilibrate with the translational temperature and in the process freezes; the translational temperature continues to decrease further since it responds to the expansion process rapidly. The level of freezing of the vibrational temperature is affected by the presence of the particles. It is also clear from this figure that the vibrational temperature freezes at a higher level than that for a pure gas case as the loading ratio is increased. This is possible since energy is transferred to gas from particles by means of heat transfer. All these phenomena can be observed from Fig. 2 also, where a larger value of χ is selected. In both the figures the broken lines represent the flow variables when no particles are present. Hence, the effect of the presence of the particles could easily be seen from these two figures. The overall physical phenomena could thus be understood from the results presented.

Conclusions

Nozzle flow of high-temperature gas-particle mixtures has been studied in this investigation. It is shown using the transformation technique reported earlier that important parameters representing the gas and particle characteristics can be grouped into two different correlating parameters and the results are self-similar for different parametric conditions of the problem. The results show that the vibrational temperature freezes at a higher level as particle loading ratio is increased. The effect of the size of the particles on the microscopic properties of the gas is revealed by means of varying the correlating parameter χ. The combined effect of size and loading ratio of the particles on the vibrational relaxation phenomenon revealed in this study is important in understanding the flow behavior in nozzles. Another important conclusion is that the variation in χ represents the complete (phase) nonequilibrium regime of gas-particle flow from near equilibrium to near-frozen cases.

Acknowledgment

R. K. Thulasiram gratefully acknowledges the Council of Scientific and Industrial Research, Government of India for the financial support made available to him through the award of Senior Research Fellowship in Engineering.

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Effects of Blowing/Suction on Vortex Instability of Horizontal Free Convection Flows

Jiin-Yuh Jang* and Kun-Nun Lie†
National Cheng-Kung University, Tainan 70101, Taiwan, Republic of China

Introduction

THE problems of the vortex mode of instability in laminar free convection flow over a heated plate in a viscous fluid have been studied extensively. The instability mechanism is due to the presence of a buoyancy force component in the direction normal to the plate surface. The appearances of longitudinal vortices were observed by Sparrow and Husar.[1] Linear stability analyses were performed by Hwang and Cheng,[2] Haaland and Sparrow,[3] Kahawita and Meroney,[4] and Chen et al.[5–7] In these analyses, a quasiparallel flow model is assumed wherein the streamwise dependence of the basic flow is not neglected, but the disturbances are assumed to be independent of the streamwise direction. Recently, Lee et al.[8,9] applied a nonparallel flow model in which the streamwise dependence of the disturbance amplitude functions is taken into account, to re-examine the vortex instability of natural convection boundary-layer flows. The nonparallel flow analysis provides a larger critical Grashof number than the quasiparallel analysis, thus bringing the prediction closer to available experimental data.

All of the works mentioned above are only for flows over an impermeable surface. Free convection with blowing and suction over a vertical plate was studied.[10–12] Recently, Lee and Hsu[13] investigated the effects of blowing or suction on mixed convection over an inclined plate. However, the influence of blowing and suction on the flow and vortex instability of free convection boundary-layer flow over a horizontal surface does not seem to have been investigated. This has motivated the present investigation.

Received Sept. 14, 1992; revision received Feb. 16, 1993; accepted for publication Feb. 16, 1993.

*Professor, Department of Mechanical Engineering.
†Graduate Assistant.

Mathematical Formation

Base Flow

Consider a semi-infinite horizontal permeable surface with uniform temperature T_w and nonuniform surface mass flux v_w. The x coordinate represents the distance along the plate from its leading edge, and the y coordinate the distance normal to the surface. The surface mass flux is assumed to be a power function of x, i.e., $v_w = Ax^m$, where A and m are constants with $A > 0$ for blowing and $A < 0$ for suction. Under the assumption of constant fluid properties, along with application of the Boussinesq and boundary-layer approximations, the laminar governing equations can be written as[5]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta\frac{\partial}{\partial x}\int_y^\infty (T - T_\infty)\,\mathrm{d}y + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u and v are the velocity components in the x and y directions; T is the temperature; β, α, and ν are the coefficients of thermal expansion, thermal diffusivity, and kinematic viscosity of the fluid; and g is the gravitational acceleration.

The boundary conditions for this problem are

$$\begin{aligned} &x = 0; && T = T_\infty, \quad u = 0, \quad v = 0 \\ &x > 0,\ y = 0; && T = T_w, \quad u = 0, \quad v = v_w = Ax^m \\ &y \to \infty; && T = T_\infty, \quad u = 0 \end{aligned} \tag{4}$$

On introducing the following transformation:

$$\eta = \frac{y}{x}\left(\frac{Gr_x}{5}\right)^{1/5}, \quad f(\eta) = \frac{\psi(x,y)}{5\nu(Gr_x/5)^{1/5}}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where ψ is the stream function which automatically satisfies the continuity Eq. (1), and $Gr_x = [g\beta(T_w - T_\infty)x^3]/\nu^2$ is the local Grashof number. Equations (1–3) can be nondimensionalized as follows:[5]

$$f''' + 3ff'' - (f')^2 + \frac{2}{5}\left(\eta\theta + \int_\eta^\infty \theta\,\mathrm{d}\eta\right) = 0 \tag{6}$$

$$\frac{1}{Pr}\theta'' + 3f\theta' = 0 \tag{7}$$

with the boundary conditions

$$\theta(0) = 1, \quad f'(0) = 0, \quad f(0) = f_w = \frac{-Ax^{m+0.4}}{3\nu\left[\dfrac{g\beta(T_w - T_\infty)}{5\nu^2}\right]^{1/5}}$$

$$\theta(\infty) = 0, \quad f'(\infty) = 0 \tag{8}$$